Vector Algebra

Question1

Three vectors each of magnitude $3\sqrt{1.5}$ units are acting at a point. If the angle between any two vectors is $\frac{\pi}{3}$, then the magnitude of the resultant vector of the three vector is

AP EAPCET 2025 - 26th May Morning Shift

Options:

A.

$9\sqrt{3}$ units

B.

9 units

C.

$\sqrt{6}$ units

D.

3 units

Answer: B

Solution:

Given data

Each vector has magnitude $a = 3\sqrt{1.5} = 3\sqrt{\frac{3}{2}}$.

Angle between any pair of vectors $= \frac{\pi}{3}$.

Let the three vectors be $\vec{A}, \vec{B}, \vec{C}$.

We want the magnitude of $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

Step 1: Compute $R^2 = |\vec{A} + \vec{B} + \vec{C}|^2$

$R^2 = A^2 + B^2 + C^2 + 2(\vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{C} + \vec{C} \cdot \vec{A})$

Since all magnitudes are equal:

$A^2 = B^2 = C^2 = a^2,$

and

$\vec{A} \cdot \vec{B} = a^2 \cos\left(\frac{\pi}{3}\right) = a^2\left(\frac{1}{2}\right).$

Step 2: Substitute

$R^2 = 3a^2 + 2 \times 3 \times a^2\left(\frac{1}{2}\right) = 3a^2 + 3a^2 = 6a^2.$

Therefore,

$R = a\sqrt{6}.$

Step 3: Substitute value of a

$a = 3\sqrt{1.5} = 3\sqrt{\frac{3}{2}}.$

So,

$R = 3\sqrt{\frac{3}{2}} \times \sqrt{6} = 3\sqrt{\frac{3\times6}{2}} = 3\sqrt{9} = 3 \times 3 = 9.$

✅ **Final Result:**

$\boxed{R = 9 \text{ units}}$

Correct Option: B) 9 units

Question2

A vector perpendicular to the vector $(4\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$ is

AP EAPCET 2025 - 26th May Morning Shift

Options:

A.

$4\hat{i}+3\hat{j}$

B.

$6\hat{i}$

C.

$3\hat{i}-4\hat{j}$

D.

$7\hat{\mathbf{k}}$

Answer: D

Solution:

To find a vector that is perpendicular to $(4\hat{\mathbf{i}}-3\hat{\mathbf{j}})$, remember that two vectors are perpendicular if their dot product is zero.

Let's check with $7\hat{\mathbf{k}}$. Here, $\hat{\mathbf{k}}$ is perpendicular to both $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

Compute the dot product: $(4\hat{\mathbf{i}}-3\hat{\mathbf{j}})\cdot 7\hat{\mathbf{k}}=4\times 0-3\times 0+0\times 7=0$.

So, $7\hat{\mathbf{k}}$ is perpendicular to $(4\hat{\mathbf{i}}-3\hat{\mathbf{j}})$.

Question3

If α,β and γ are the angles made by a vector with x,y and z axes respectively, then $\sin^2\alpha+\sin^2\beta=$

AP EAPCET 2025 - 23rd May Evening Shift

Options:

A.

$\sin^2\gamma$

B.

$\cos^2\gamma$

C.

$1+\cos^2\gamma$

D.

$1+\sin^2\gamma$

Answer: C

Solution:

We know that,

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow 1-\sin^2\alpha+1-\sin^2\beta+\cos^2\gamma=1$

$\Rightarrow 1+\cos^2\gamma-\left(\sin^2\alpha+\sin^2\beta\right)=0$

$\Rightarrow 1+\cos^2\gamma=\sin^2\alpha+\sin^2\beta$

$\Rightarrow \sin^2\alpha+\sin^2\beta=1+\cos^2\gamma$

Question4

If the magnitude of a vector $\mathbf{P}$ is 25 units and its y-component is 7 units, then its x-component is

AP EAPCET 2025 - 22nd May Evening Shift

Options:

A.

24 units

B.

18 units

C.

32 units

D.

16 units

Answer: A

Solution:

$P=|P|=25$ units

$P_y=7$

Since, $\sqrt{P_x^2+P_y^2}=P$

$$\Rightarrow \quad P_x^2+P_y^2=P^2$$
$$\Rightarrow \quad P_x^2+7^2=25^2$$
$$\Rightarrow \quad P_x=\sqrt{25^2-7^2}$$
$$=\sqrt{625-49}$$
$$=\sqrt{576}=24 \text{ units}$$

Question5

The magnitudes of two vectors are A and $B(A>B)$. If the maximum resultant magnitude of the two vectors is ' n ' times their minimum resultant magnitude, then $\frac{A}{B}=$

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

$\frac{n}{n-1}$

B.

$\frac{n+1}{n}$

C.

$\frac{n^2+1}{n-1}$

D.

$\frac{n+1}{n-1}$

Answer: D

Solution:

We know that,

$$
\begin{aligned}
& R_{\max }=A+B \text { and } R_{\min }=A-B \\
& \text {Since, } R_{\max }=n R_{\min } \\
& \Rightarrow \quad A+B=n(A-B) \\
& \Rightarrow \quad \frac{A+B}{A-B}=\frac{n}{1}
\end{aligned}
$$

Applying componendo and dividendo rule,

$$
\begin{aligned}
& \frac{(A+B)+(A-B)}{(A+B)-(A-B)}=\frac{n+1}{n-1} \\
\Rightarrow \quad & \frac{2 A}{2 B}=\frac{n+1}{n-1} \Rightarrow \frac{A}{B}=\frac{n+1}{n-1}
\end{aligned}
$$

Question6

The angle made by the resultant vector of two vectors $2 \hat{\mathbf{i}}+3 \hat{\mathbf{j}}+4 \hat{\mathbf{k}}$ and $2 \hat{\mathbf{i}}+7 \hat{\mathbf{j}}-4 \hat{\mathbf{k}}$ with X-axis.

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. 60°

B. 45°

C. 90°

D. 120°

Answer: B

Solution:

The task is to determine the angle the resultant vector of two given vectors makes with the X-axis.

Given vectors are:

$$
\begin{aligned}
& \mathbf{A}=2 \hat{\mathbf{i}}+3 \hat{\mathbf{j}}+4 \hat{\mathbf{k}} \\
& \mathbf{B}=2 \hat{\mathbf{i}}-7 \hat{\mathbf{j}}-4 \hat{\mathbf{k}}
\end{aligned}
$$

First, calculate the resultant vector $\mathbf{R}$:

$$\begin{aligned}\mathbf{R} &= \mathbf{A} + \mathbf{B} \\ &= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - 4\hat{\mathbf{k}}) \\ &= (2+2)\hat{\mathbf{i}} + (3-7)\hat{\mathbf{j}} + (4-4)\hat{\mathbf{k}} \\ &= 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 0\hat{\mathbf{k}} \\ &= 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\end{aligned}$$

To find the angle θ between the resultant vector $\mathbf{R}$ and the X-axis, we use the dot product:

$$\cos(\theta) = \frac{\mathbf{R}\cdot\hat{\mathbf{i}}}{|\mathbf{R}|\cdot|\hat{\mathbf{i}}|}$$

Calculate the dot product $\mathbf{R} \cdot \hat{\mathbf{i}}$:

$$\mathbf{R} \cdot \hat{\mathbf{i}} = (4\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) \cdot \hat{\mathbf{i}} = 4$$

Calculate the magnitude of $\mathbf{R}$:

$$|\mathbf{R}| = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

Now, substitute these values into the cosine equation:

$$\begin{aligned}\cos\theta &= \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} \\ \theta &= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) \\ \theta &= 45^\circ\end{aligned}$$

Therefore, the resultant vector makes an angle of 45° with the X-axis.

Question7

If $|\mathbf{P} + \mathbf{Q}| = |\mathbf{P}| = |\mathbf{Q}|$ then the angle between $\mathbf{P}$ and $\mathbf{Q}$ is

AP EAPCET 2024 - 20th May Morning Shift

Options:

A. 0°

B. 120°

C. 60°

D. 90°

Answer: B

Solution:

Given that $|\mathbf{P}+\mathbf{Q}|=|\mathbf{P}|=|\mathbf{Q}|$, let's analyze the problem step-by-step:

Begin with the equation:

$$|\mathbf{P}+\mathbf{Q}|^2=|\mathbf{P}|^2$$

Expanding $(\mathbf{P}+\mathbf{Q})\cdot(\mathbf{P}+\mathbf{Q})$:

$$(\mathbf{P}+\mathbf{Q})\cdot(\mathbf{P}+\mathbf{Q})=\mathbf{P}\cdot\mathbf{P}+2\mathbf{P}\cdot\mathbf{Q}+\mathbf{Q}\cdot\mathbf{Q}=P^2+2\mathbf{P}\cdot\mathbf{Q}+Q^2$$

Since $|\mathbf{P}|=|\mathbf{Q}|$, let them both be P (i.e., $Q=P$). Thus, the equation becomes:

$$P^2+2\mathbf{P}\cdot\mathbf{Q}+P^2=P^2$$

Simplify and rearrange:

$$2\mathbf{P}\cdot\mathbf{Q}+P^2=0$$

Substitute $\mathbf{P}\cdot\mathbf{Q}=PQ\cos\theta$:

$$2PQ\cos\theta+P^2=0$$

Solve for $\cos\theta$:

$$2P^2\cos\theta=-P^2$$

$$\cos\theta=-\frac{1}{2}$$

Recognize that $\cos\theta=-\frac{1}{2}$ corresponds to:

$$\theta=120^\circ$$

Thus, the angle between $\mathbf{P}$ and $\mathbf{Q}$ is 120°.

Question8

The component of a vector $\mathbf{P}=3\hat{i}+4\hat{j}$ along the direction $(\hat{i}+2\hat{j})$ is

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. $\frac{8}{\sqrt{5}}$

B. $\frac{11}{\sqrt{5}}$

C. $\frac{11}{2}$

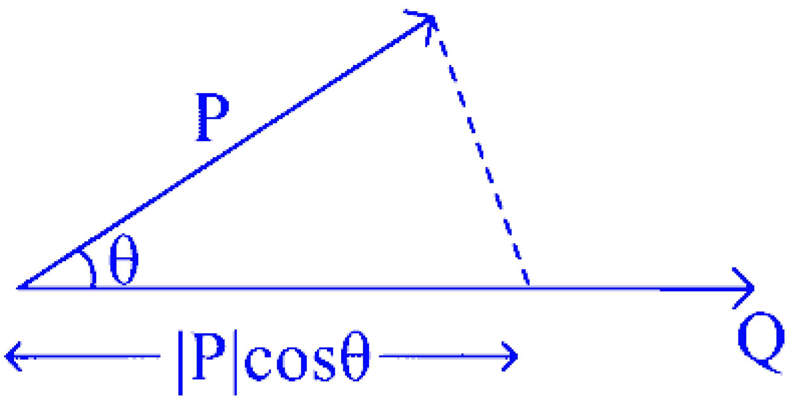
D. $\sqrt{10}$

Answer: B

Solution:

Given, $\mathbf{P}=3\hat{i}+4\hat{j}$

Let $Q=\hat{i}+2\hat{j}$



The component of $\mathbf{P}$ along the $\mathbf{Q}$ is given as $|\mathbf{P}|\cos\theta$.

Hence, we know that,

$$\begin{aligned}
&\text{Hence, } \mathbf{P}\cdot\mathbf{Q}=|\mathbf{P}||\mathbf{Q}|\cos\theta \\
&\Rightarrow |P|\cos\theta=\frac{\mathbf{P}\cdot\mathbf{Q}}{|\mathbf{Q}|} \\
&\qquad\qquad =\frac{(3\hat{i}+4\hat{j})\cdot(\hat{i}+2\hat{j})}{\sqrt{1^2+2^2}}=\frac{3+8}{\sqrt{5}}=\frac{11}{\sqrt{5}}
\end{aligned}$$

Question9

If two vectors $\mathbf{A}$ and $\mathbf{B}$ are mutually perpendicular, then the component of $\mathbf{A}-\mathbf{B}$ along the direction of $\mathbf{A}+\mathbf{B}$ is

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. $\sqrt{|A|^2+|B|^2}$

B. $\sqrt{|A|^2-|B|^2}$

C. $\frac{|A|^2-|B|^2}{\sqrt{|A|^2+|B|^2}}$

D. $\frac{|A|^2+|B|^2}{\sqrt{|A|^2-|B|^2}}$

Answer: C

Solution:

The two vectors $\mathbf{A}$ and $\mathbf{B}$ are mutually perpendicular to each other

$$\begin{aligned} & \mathbf{A} \cdot \mathbf{B}=0 \\ \Rightarrow \quad & A B \cos \theta=0 \\ \Rightarrow \quad & \cos \theta=0 \Rightarrow \theta=90^{\circ} \\ |\mathbf{A}-\mathbf{B}| & \\ & =\sqrt{A^2+B^2-2 A B \cos 90^{\circ}} \\ & =\sqrt{A^2+B^2} \end{aligned}$$

Unit vector along $\mathbf{A}-\mathbf{B}$

$=\frac{\mathbf{A}-\mathbf{B}}{|\mathbf{A}-\mathbf{B}|}=\frac{\mathbf{A}-\mathbf{B}}{\sqrt{A^2+B^2}}$

component of $(\mathbf{A}+\mathbf{B})$ in direction of $(\mathbf{A}-\mathbf{B})$ will be given as,

$$\begin{aligned} & =\frac{(\mathbf{A}+\mathbf{B}) \cdot(\mathbf{A}-\mathbf{B})}{|\mathbf{A}-\mathbf{B}|} \\ & =\frac{\mathbf{A} \cdot \mathbf{A}-\mathbf{A} \cdot \mathbf{B}+\mathbf{B} \cdot \mathbf{A}-\mathbf{B} \cdot \mathbf{B}}{\sqrt{A^2+B^2}} \\ & =\frac{A^2-B^2}{\sqrt{A^2+B^2}}=\frac{|\mathbf{A}|^2-|\mathbf{B}|^2}{\sqrt{|\mathbf{A}|^2+|\mathbf{B}|^2}} \end{aligned}$$

Question10

Which of the following is not true about vectors $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$?

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. $(\mathbf{A} \cdot \mathbf{A})(\mathbf{B} \cdot \mathbf{C})$ is a scalar value.

B. $(\mathbf{A} \times \mathbf{B}), (\mathbf{B} \times \mathbf{C})$ is a scalar value.

C. $(\mathbf{A} \times \mathbf{C}) \times (\mathbf{B} \times \mathbf{C})$ is a scalar value.

D. $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ is a vector value.

Answer: C

Solution:

In option (a)
(A. A) (B. Q)
$\left(A^2\right)(BC \cos \theta)$
$= A^2 BC \cos \theta$ Scalar value

In option (b)

$$\begin{aligned}(\mathbf{A} \times \mathbf{B})(\mathbf{B} \times \mathbf{Q} &= (AB \sin \theta \hat{\mathbf{n}}) \cdot (BC \sin \phi \hat{k}) \\ &= (AB \sin \theta)(BC \sin \phi)(\hat{\mathbf{n}} \cdot \hat{k})\end{aligned}$$

where, $\hat{\mathbf{n}}$ is unit vector perpendicular to both vector $\mathbf{A}$ and $\mathbf{B}$ and $\hat{k}$ is unit vector perpendicular to both vector $\mathbf{B}$ and $\mathbf{C}$.

$\therefore \hat{\mathbf{n}}. \hat{k} =$ scalar value

$\therefore (\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{B} \times \mathbf{Q}$ is a scalar valueIn option (C)

$$\left(\mathbf{A} \times \mathbf{Q} \times \left(\mathbf{B} \times \mathbf{Q} = (AC \sin \theta_1)\hat{\mathbf{r}} \times (BC \sin \theta_2)\hat{\mathbf{r}}_2\right.\right.$$
$$= (AC \sin \theta_1)\,(BC \sin \theta_2)\,\left(\hat{\mathbf{r}} \times \hat{\mathbf{r}}_2\right)$$

$\hat{\mathbf{r}}_1 \times \hat{\mathbf{r}}_2$ is a vector product, hence it gives a vector value.

Therefore, $\mathbf{A} \times \mathbf{Q} \times (\mathbf{B} \times \mathbf{Q}$ is a vector value.

In option (d) $\mathbf{A} \times (\mathbf{B} \times \mathbf{Q}$ is vector triple product which gives a vector value.

Question11

One of the rectangular components of a forceof 40 N is $20\sqrt{3}$ N. What is the otherrectangular component?

AP EAPCET 2021 - 19th August Morning Shift

Options:

A. 10 N

B. 20 N

C. 30 N

D. 25 N

Answer: B

Solution:

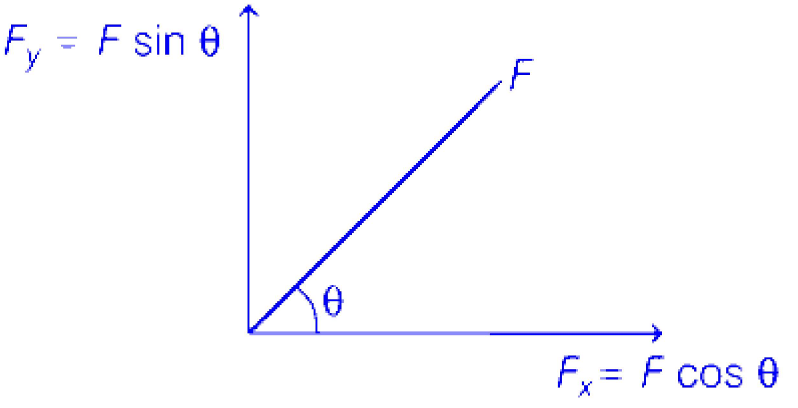
Given, applied force, F = 40 N

Horizontal component of force, $F_x = 20\sqrt{3}$

and θ be the angle of force from horizontal X-axis.

According to given diagram,

The component of F along X-axis will be



$$F_x = F\cos\theta$$

$$\Rightarrow 20\sqrt{3} = 40\cos\theta \Rightarrow \sqrt{3}/2 = \cos\theta$$

$$\therefore \quad \theta = \cos^{-1}(\sqrt{3}/2) = 30^\circ$$

And the component of F along Y-axis

$F_y = F\sin\theta$

$= F\sin 30^\circ$

$= 40 \times 1/2 = 20$ N
